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A nonparametric method for the measurement of size diversity with emphasis on data standardization

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Web Appendix A

Diversity of some parametric families—Diversity index can be exactly obtained for some simple densities. Some of these results can be found in Johnson (2004, Appendix A), for instance, uniform, normal, log-normal, exponential, and generalized Pareto distributions. The set function $I\{\cdot\}$ is used to define the support of the variables; its value is 1 whenever the condition in the argument is true and 0 elsewhere. These results are as follows:

Uniform distribution, U(a, b)—The uniform pdf and its diversity are

$$p_U(x) = \frac{1}{b-a} I\left\{a \le x \le b\right\}, \mu(U) = \ln(b-a).$$

The uniform distribution plays a role of reference as a maximum diversity, uncertainty, or homogeneity on a finite interval. It can be used as reference of diversities defined in a finite interval. For instance, if $p_X(x)$ is some probability density with support (a, b), the evenness is defined as the ratio $\mu(X)/\mu(U) = \mu(X)/\ln(b - a)$, which has values less than one.

Normal distribution, N(m, σ²)—The normal pdf and its diversity are

$$p_N(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right], \mu(N) = \frac{1}{2} + \ln(\sqrt{2\pi}\sigma)$$

Note that $\mu(N)$ does not depend on the mean m (shift-invariance). For random variables whose variance is σ^2, $\mu(N)$ is the maximum diversity attainable.

Exponential distribution, Exp(λ)—The exponential pdf and its diversity are

$$P_E(x) = \lambda \exp\left[-\lambda x\right] I\left\{0 < x < +\infty\right\}, \mu(E) = 1 - \ln(\lambda)$$

Log-Normal distribution, $LN(m_{\ln}, \sigma^2_{\ln})$—The Log-Normal pdf and its diversity are

$$p_{LN}(x) = \frac{1}{x\sqrt{2\pi}\sigma_{ln}} \exp\left[-\frac{(\ln x - m_{\ln})^2}{2\sigma^2_{\ln}}\right], \mu(LN) = \frac{1}{2} + \ln(\sqrt{2\pi}\sigma_{\ln}) + m_{\ln}$$

where $\exp(m_{\ln})$ is the median and $\sigma^2_{\ln}$ is the logarithmic variance.

Generalized Pareto distribution, GPD(ξ,β)—The pdf of the *GPD*(ξ,β) is

$$p_{GPD}(x) = \frac{1}{\beta}\left(1 + \frac{\xi}{\beta}x\right)^{-(1+\xi)/\xi} I\left\{0 < x < x_{\sup}\right\} \quad (12)$$

where $\beta > 0$ and $x_{\sup} = +\infty$ if $\xi > 0$, and $x_{\sup} = -\beta/\xi$ if $\xi < 0$. If $\xi = 0$, the limiting form is an exponential density with parameter $\lambda = \beta^{-1}$. The associated diversity index, for all values of ξ, is

$$\mu(\mathrm{GDP}) = 1 + \ln \beta + \xi$$

in agreement to the diversity index of an exponential distribution obtained by setting $\xi = 0$.