COMMENTS

Effect of salt content on the temperature of maximum density and on static stability in Lake Ontario

Osborn and LeBlond (1974) examined the static stability of freshwater lakes by calling attention to the adiabatic effects on the calculation. They arrived at the following criterion for stability

$$(\partial V/\partial T)_P[dT/dz-(\partial T/\partial z)_\theta]>0. \quad (1)$$

They converted Eq. 1 to

$$(T-T_{MD})[dT/dz-(\partial T/\partial z)_\theta]>0, \quad (2)$$

where V is the specific volume, T the in situ temperature in degrees Celsius, T_{MD} the temperature of maximum density, P the pressure, θ the entropy, and z is the height which increases upward. dT/dz and $(\partial T/\partial z)_\theta$ are the in situ temperature gradient and adiabatic temperature gradient. The adiabatic temperature gradient is calculated from the equation

$$(\partial T/\partial z)_\theta=\frac{(T+273.16)(\partial V/\partial T)_P}{Cp}\frac{\partial P}{\partial z}, \quad (3)$$

where Cp is the heat capacity at constant pressure.

Osborn and LeBlond used Eq. 2 to evaluate the static stability without considering the salt content of the lake; however, Lake Ontario has a rather high salt content (about 185 ppm: Beeton 1965; about 235 ppm: Dobson 1967). The average salt content is roughly 0.209‰ in salinity, which is enough to depress the temperature of maximum density by more than 0.04°C (calculated from the equation of state:

Table 1. Vertical temperature structure in Lake Ontario.

		T_{MD}*			$(\partial T/\partial z)_\theta$†	
Depth (m)	Temp (°C)	S = 0 (°C)	S = 0.209°/oo (°C)	Avg dT/dz (°C m^{-1})	S = 0 (°C m^{-1})	S = 0.209°/oo (°C m^{-1})
28	4.45	3.928	3.881		5.5x10^{-6}	6.0x10^{-6}
				1.8x10^{-1}		
31	3.91	3.922	3.875		-1.2x10^{-7}	3.8x10^{-7}
				1.7x10^{-2}		
34	3.86	3.916	3.869		-5.8x10^{-7}	-8.6x10^{-8}
				-2.3x10^{-2}		
37	3.93	3.910	3.864		2.1x10^{-7}	7.1x10^{-7}
				-3.3x10^{-3}		
40	3.94	3.904	3.858		3.8x10^{-7}	8.8x10^{-7}
				4.0x10^{-3}		
50	3.90	3.884	3.837		1.7x10^{-7}	6.7x10^{-7}
				4.0x10^{-3}		
75	3.80	3.834	3.787		-3.5x10^{-7}	1.5x10^{-7}
				2.8x10^{-3}		
100	3.73	3.784	3.737		-5.6x10^{-7}	-6.3x10^{-8}
				1.0x10^{-3}		
138	3.69	3.708	3.661		-1.8x10^{-7}	3.2x10^{-7}

*Calculated from Chen and Millero's (1976) equation of state, in which the pure water equation was taken from the work of Fine and Millero (1973).
†Calculated from Eq. 3 and Chen and Millero's (1976) equation of state.

Chen and Millero 1976). As long as the in situ temperatures are high, this change in the temperature of maximum density does not affect Osborn and LeBlond's stability calculations. However, when the in situ temperature drops close to the region of T_{MD}, the calculation of stability is greatly influenced by the way T_{MD} is calculated.

Table 1 contains a vertical temperature structure from a survey on Lake Ontario (station 24: Limnol. Data Rep. 1. Lake Ontario. 1967. CCIW, Burlington, Ontario). The third and fourth columns in Table 1 show the temperature of maximum density at $S = 0$ and $S = 0.209‰$. The sixth and seventh columns show the values of the adiabatic temperature gradient at the corresponding salinities.

The water at a depth of 31 m will be used as an example. The water is stable if the salt content ($S = 0.209‰$) is taken into account, since $dT/dz - (\partial T/\partial z)_\theta > 0$, $T - T_{MD} > 0$, and $(T - T_{MD})[dT/dP - (\partial T/\partial P)_\theta] > 0$. Therefore, the water is stable relative to the layer immediately above. Whereas if the water is considered as "fresh" ($S = 0$), then $dT/dz - (\partial T/\partial z)_\theta > 0$ and $T - T_{MD} < 0$. In this case we arrive at the wrong conclusion because $(T - T_{MD})[dT/dz - (\partial T/\partial z)_\theta] < 0$. From the above example, it is clear that the salt content should be considered for cold water stability calculations.

A better way to calculate the vertical stability for natural waters is to calculate the Brunt-Väisälä frequency

$$N^2 = \frac{-g^2}{V^2}\left\{\left(\frac{\partial V}{\partial S}\right)_P \frac{dS}{dP} + \left(\frac{\partial V}{\partial T}\right)_P \cdot \left[\frac{dT}{dP} - \left(\frac{\partial T}{\partial P}\right)_\theta\right]\right\} > 0, \qquad (4)$$

where N is the Brunt-Väisälä frequency, g the gravitational constant, and S the salt content. Equation 4 shows that the effect of the vertical variation of salt content on stability could also be important. In the case of lakes that have no vertical salt variation, Eq. 4 reduces to Eq. 1. Equation 1 is similar to the expression of Lee and Rodgers (1974).

In conclusion, the effect of salt content on the specific volume of natural water is significant. This is especially true in cold water lakes, or in deep water lakes where the water temperatures approach the temperature of maximum density. We suggest that the vertical distribution of salt be measured, and that the Brunt-Väisälä frequency be calculated when there is a need for accurate examination of vertical stability.

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References

Beeton, A. M. 1965. Eutrophication of the St. Lawrence great lakes. Limnol. Oceanogr. **10**: 240–254.

Chen, C. T., and F. J. Millero. 1976. The high pressure specific volume of seawater. Deep-Sea Res. **23**: 595–612.

Dobson, H. H. 1967. Principal ions and dissolved oxygen in Lake Ontario. Proc. Conf. Great Lakes Res. **1967**: 337–356.

Fine, R. A., and F. J. Millero. 1973. Compressibility of water as a function of temperature and pressure. J. Chem. Phys. **59**: 5529–5536.

Lee, A. H., and G. K. Rodgers. 1974. Comments on "Static stability in freshwater lakes" (T. R. Osborn and P. H. LeBlond). Limnol. Oceanogr. **19**: 546.

Osborn, T. R., and P. H. LeBlond. 1974. Static stability in freshwater lakes. Limnol. Oceanogr. **19**: 544–545.